MATH 1A − QUIZ 11 − SOLUTIONS

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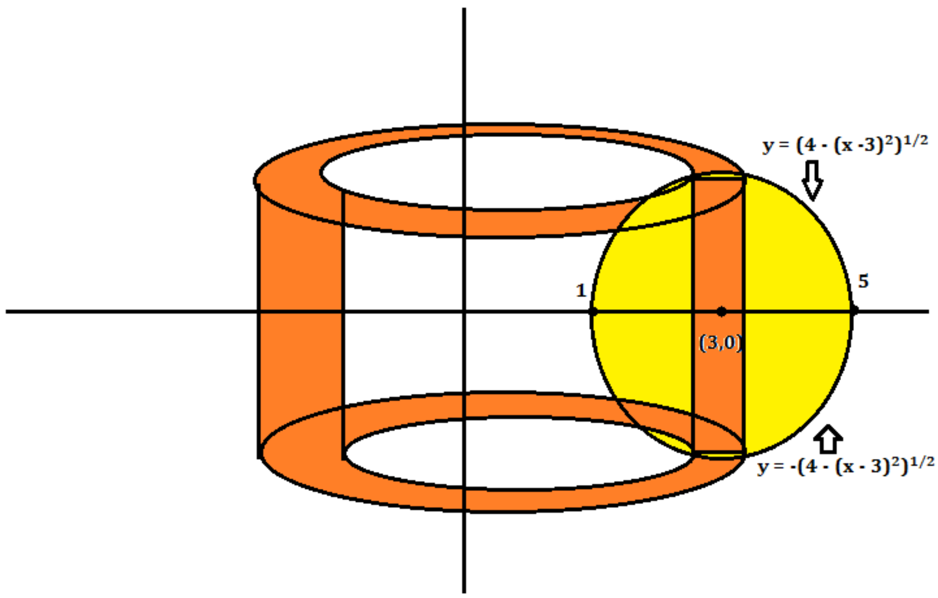
(1) *(5 points)* Find the volume of the donut obtained by rotating the disk of center $(3,0)$ and radius 2 about the $y-$axis.

1) Picture:

1A/Math 1A - Fall 2013/Quizzes/Quiz11Donut.png



2) Let's use the shell method! $x = 0$, so $k = 0$, and Radius $= |x - 0| = |x| = x$. Also, the equation of the circle is $(x-3)^2 + y^2 = 4$, so $y = \pm\sqrt{4-(x-3)^2}$, and Outer $= \sqrt{4-(x-3)^2}$ and Inner $= -\sqrt{4-(x-3)^2}$, so Height = Outer - Inner = $2\sqrt{4-(x-3)^2}$.

Hence:

$$V = \int_1^5 2\pi x\left(2\sqrt{4-(x-3)^2}\right)dx = \int_1^5 4\pi x\sqrt{4-(x-3)^2}dx$$

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3) To evaluate that integral, let $u = x - 3$, then $du = dx$, $x = u + 3$, and $u(1) = -2$, $u(5) = 2$, so:

$$V = \int_{-2}^{2} 4\pi(u+3)\sqrt{4-u^2}du = 4\pi \int_{-2}^{2} u\sqrt{4-u^2}du + 12\pi \int_{-2}^{2} \sqrt{4-u^2}du$$

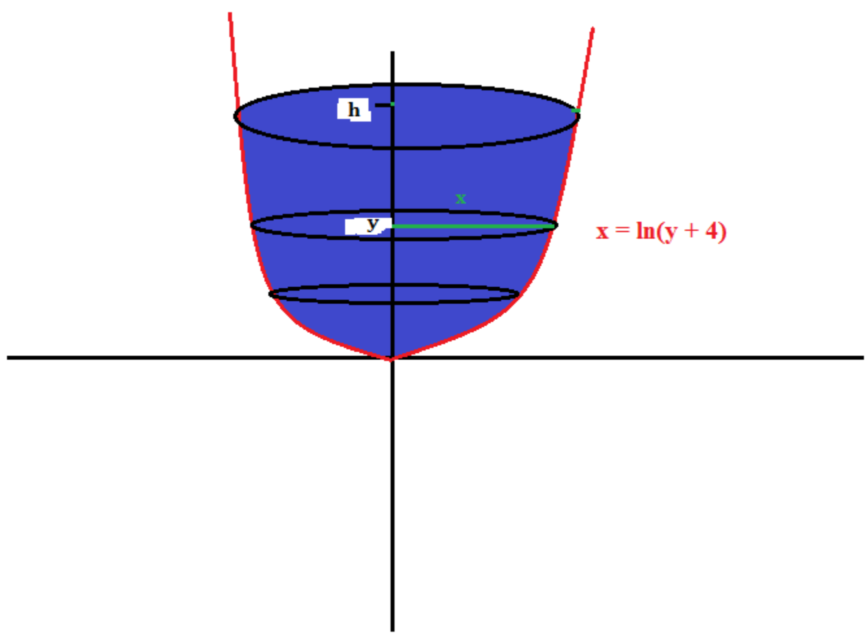
However, the first integral is 0 because $u\sqrt{4-u^2}$ is an odd function, and the second integral is $\frac{1}{2}\pi 2^2 = 2\pi$, because it represents the area of a semicircle of radius 2!

Hence, we get:

$$V = 0 + 12\pi(2\pi) = 24\pi^2$$

(2) *(5 points)* The surface of a bowl is obtained by rotating the curve $x = \ln(y+4)$, for $y \geq 0$, about the $y-$ axis. Water pours into the bowl at a constant rate of 3 cubic units/min. How fast is the water level rising when the water is $e^3 - 4$ units deep?

1) First draw a good picture of the situation:
1A/Math 1A - Fall 2013/Quizzes/Quiz11Volume.png



2) We want to find $\frac{dh}{dt}$ when $h = e^3 - 4$.

3) Using the disk method (the horizontal slices are disks), we get that:

$$V = \int_0^h \pi x^2 dy = \int_0^h \pi \left(\ln(y+4)\right)^2 dy$$

4) Differentiating the above with respect to t (using the FTC Part I and the chain rule), we get:

$$\frac{dV}{dt} = \frac{dh}{dt}\pi(\ln(h+4))^2$$

But $\frac{dV}{dt} = 3$, and $h = e^3 - 4$, so $\ln(h+4)^2 = \left(\ln(e^3)\right)^2 = 3^2 = 9$, and :

$$3 = \frac{dh}{dt}(9\pi)$$

$$\frac{dh}{dt} = \frac{1}{3\pi}\text{units/min}$$